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Optimal Guidance with Reduced Sensitivity to Time-to-Go Estimation Errors

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An optimal guidance method that exhibits reduced sensitivity to the error in time-to-go estimation is presented. Estimation of target acceleration is not required under this scheme. The gains of the reduced-sensitivity guidance law are given in terms of the inversion of a 3 × 3 matrix, the entries of which are expressed in closed form. The performance of this new guidance method is compared (via simulations) to the minimum-effort-guidance law and to proportional navigation for various error sources that affect the miss distance and as a function of the time-to-go estimation error.

I. Introduction

THE method of proportional navigation[1] is widely used for interception of nonmaneuvering targets. In cases in which target maneuvers are significant, extensions of this method are used such as augmented proportional navigation,[2] where the commanded interceptor's acceleration n_c depends on the target acceleration n_T, so that

$$n_c = \frac{N'(y + \dot{y}t_{go})}{t_{go}^2} + \frac{N'}{2}n_T \tag{1}$$

where y is the relative interceptor–target separation, t_{go} is the time-to-go until intercept, and N' is the so-called navigation constant. Note that the line-of-sight-rate is given by

$$\dot{\lambda} = \frac{y + \dot{y}t_{go}}{V_c(t_{go})^2} \tag{2}$$

where V_c is the closing velocity. The first term of Eq. (1) can easily be mechanized given a gimbaled seeker, which measures the line-of-sight rate $\dot{\lambda}$. For the second term, an estimation of the target acceleration n_T is required.

However, if the transfer function relating the commanded and actual accelerations n_c and n_L has a significant time lag (with respect to the final time t_f) the augmented proportional navigation law can lead to a significant miss distance. To compensate for this, one can employ the minimum effort guidance law (MEL), which minimizes

$$J = \int_0^{t_f} n_c^2(t)\,dt$$

subject to state equations that include the time-lag effect and to $y(t_f) = 0$. It is well known that proportional navigation is the optimal strategy obtained with this cost functional for an ideal pursuer against a nonmaneuvering target.[2] The resulting guidance law nicely overcomes the large time-lag problem but is strongly dependent on the time constant and the time-to-go, namely

$$n_c = \frac{N'(y + \dot{y}t_{go})}{t_{go}^2} + \frac{N'}{2}n_T - n_L K_L \tag{3}$$

where

$$N' = \frac{6h^2(e^{-h} - 1 + h)}{2h^3 + 3 + 6h - 6h^2 - 12he^{-h} - 3e^{-2h}} \tag{4}$$

and

$$K_L = \left(N'T^2/t_{go}^2\right)(e^{-h} + h - 1) \tag{5}$$

and where

$$h = t_{go}/T \tag{6}$$

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In these equations, T is the time constant that relates the actual interceptor's acceleration and the corresponding command, namely

$$n_L/n_c = 1/(1+Ts) \tag{7}$$

where s is the differentiation operator, namely $s = d/dt$.

There are two principal ways of obtaining guidance laws based on classical proportional navigation. One way (exhibited by MEL) is to look upon proportional navigation as an optimal strategy that minimizes the control effort, whereas the other approach is to view it as a proportional feedback of the zero-effort miss.[2] The latter approach yields powerful predictive guidance methods for which a general treatment can be found in Ref. 3. In this paper, however, we proceed with the optimal control strategy.

In Ref. 4, an optimal interception problem is considered that takes into account uncertainty of the time constant T. An even more important issue is the effect of the estimation error of the time-to-go. It has been shown that such an estimation error can significantly degrade the performance of the optimal guidance law.[5]

The present paper develops a straightforward method for reducing the uncertainty in time-to-go, which follows from the observation that the terminal relative lateral velocity $\dot{y}(t_f)$ represents the sensitivity of the miss distance with respect to uncertainty in the time-to-go estimate. To show this, we can use the following result from Ref. 6, which employs a differential-game approach with terminal lateral velocity constraints. There, the guidance system was assumed to have a negligible time lag. The case of zero terminal conditions (game-theoretic optimal rendezvous) was solved analytically and the optimal strategies as well as the resulting optimal trajectories were given in closed form. The relative separation was seen to satisfy the following:

$$y(t) = c_1 t_{go}^2 + c_2 t_{go}^3 = c_1(t_f - t)^2 + c_2(t_f - t)^3$$

where t_f is the final time and c_1, c_2 are constants of integration, as opposed to $y(t) = c_1(t_f - t) + c_2(t_f - t)^2$ satisfied by the game-theoretic optimal intercept (zero miss without additional terminal constraints). Because $y(t_f)$ is the miss distance, it is clear that the latter approach is more sensitive to uncertainty in t_f than the former for which

$$\left.\frac{\partial y(t)}{\partial t_f}\right|_{t=t_f} = 0$$

By employing either proportional navigation or augmented proportional navigation with a direct measurement of $\dot{\lambda}$, the sensitivity to time-to-go errors disappears. However, these strategies, as already mentioned, completely ignore the time constant T.

The idea of sensitivity reduction is pursued in the present paper. For a single lag guidance system model, a linear quadratic differential game problem is formulated with a cost that includes, in addition to the adversary's control efforts and the miss distance, a weighted combination of the terminal lateral velocity and the pursuer's acceleration. The latter is included for the purpose of relaxing the pursuer's acceleration requirements. The problem will be solved in closed form and a numerical parametric study will be performed employing various weights to demonstrate the robustness of this method. Estimation of the target's acceleration is not required for this scheme. Finally, we compare the numerical results to the MEL strategy (which requires target acceleration) and to proportional navigation.

II. Problem Formulation

We make the following assumptions: 1) the pursuit–evasion conflict is two dimensional in the horizontal plane; 2) the speeds of the pursuer P and the evader E are constant; 3) the trajectories of P and E can be linearized around their collision course; 4) the evader directly controls its lateral acceleration and the pursuer is modeled as a first-order system; and 5) the pursuer is more maneuverable than the evader.

Assumptions 1–3, concerning the linearization around a collision triangle, are quite common. We note that in the numerical part of this work we do not consider errors caused by time-to-go estimates on the collision course itself, because tail-chase and head-on scenarios are insensitive to these errors when constructing the nominal collision course (straight lines). The analytical results, however, are not restricted to these two cases.

Assumption 4 ignores the high-order effects and transport delays of the guidance system and is, therefore, quite restrictive. However, it enables us to obtain simple analytical solutions.

The last assumption, requiring the evader to be less maneuverable than the pursuer, entails a higher penalty on its control effort in the performance index.

Under these assumptions, the problem has the following state-space representation[2]:

$$\dot{x} = Ax + Bu + Dw$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/T \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/T \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{8}$$

where x_1, x_2, and x_3, the components of the state vector x, are the relative separation, the lateral relative velocity, and the normal acceleration of the pursuer, respectively; u is the commanded acceleration of the pursuer; and w is the normal acceleration of the evader.

The minimax problem is the following:

$$\min_u \max_w J = \frac{a}{2}x_1^2(t_f) + \frac{b}{2}x_2^2(t_f) + \frac{c}{2}x_3^2(t_f) + \frac{1}{2}\int_0^{t_f}[u^2(t) - \gamma^2 w^2(t)]\,dt \tag{9}$$

The coefficient a weights the miss distance and is always positive. The coefficients b and c weight the terminal velocity and acceleration, respectively, and are nonnegative in this work. The former is used to obtain the desired sensitivity reduction to time-to-go errors, and the latter can be used to control the terminal acceleration. γ penalizes evasive maneuvers and, therefore, because of Assumption 5, is required to be $\gamma > 1$.

Note, finally, that Eqs. (3–6), with $n_T = 0$, are the solution to the minimax problem for the special case where $a \to \infty$, $\gamma \to \infty$, $b = c = 0$.

III. Optimal Feedback Solution

Linear quadratic differential games theory yields the following linear-feedback optimal strategies[7]:

$$u = -B^T P x \qquad w = D^T \gamma^{-2} P x \tag{10}$$

where the Riccati matrix P is the solution of the differential Riccati equation

$$-\dot{P} = PA + A^T P - PBB^T P + \gamma^{-2} PDD^T P \tag{11}$$

with the terminal conditions

$$P(t_f) = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Let $S \equiv P^{-1}$ and define

$$S = \begin{bmatrix} S1 & S2 & S3 \\ S2 & S4 & S5 \\ S3 & S5 & S6 \end{bmatrix}$$

Thus,[8]

$$\dot{S} = AS + SA^T - BB^T + \gamma^{-2} DD^T \tag{12}$$

and

$$S(t_f) = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{bmatrix}$$

Equation (12) can be rewritten explicitly as

$$\dot{S} = \begin{bmatrix} -2S2 & -S4 - S3 & -S5 + S3/T \\ & -2S5 - 1/\gamma^2 & -S6 + S5/T \\ & & 2S6/T + 1/T^2 \end{bmatrix} \tag{13}$$

The solution for $S6$ can be easily obtained:

$$S6(t) = -(1/2T) + e^{2h}[(1/2T) + (1/c)] \tag{14}$$

where h is defined by Eq. (6). We can continue now to solve sequentially for $S5 \cdots S1$. The following results have been obtained by MAPLE:

$$S5(t) = -\frac{1}{2} - \frac{1}{2}e^{2h} - \frac{Te^{2h}}{c} + \frac{e^h(c + T)}{c} \tag{15}$$

$$S4(t) = t_{go} + \frac{T}{2}e^{2h} + \frac{T^2}{c}e^{2h} - 2Te^h$$

$$-\frac{2T^2}{c}e^h - \frac{t_{go}}{\gamma^2} + \frac{3T}{2} + \frac{T^2}{c} + \frac{1}{b} \tag{16}$$

$$S3(t) = -\frac{T}{2} + \frac{T}{2}e^{2h} + \frac{T^2e^{2h}}{c} - e^h t_{go} - \frac{e^h T t_{go}}{c} - \frac{T^2 e^h}{c} \tag{17}$$

$$S2(t) = -\frac{t_{go}^2}{2} - \left(\frac{T^2}{2} + \frac{T^3}{c}\right)e^{2h} - \frac{T^2c + 2T^3}{2c}$$

$$+\left(T^2 - \frac{2T^3}{c} + Tt_{go} + \frac{T^2 t_{go}}{c}\right)e^h$$

$$+\frac{t_{go}^2}{2\gamma^2} - Tt_{go} - \frac{T^2 t_{go}}{c} - \frac{t_{go}}{b} \tag{18}$$

$$S1(t) = \frac{t_{go}^3}{3} + \left(\frac{T^3}{2} + \frac{T^4}{c}\right)e^{2h}$$

$$-\left(2T^2 t_{go} - \frac{2T^4}{c} - \frac{2T^3 t_{go}}{c}\right)e^h - \frac{t_{go}^3}{3\gamma^2} + Tt_{go}^2 + \frac{T^2 t_{go}^2}{c}$$

$$+\frac{t_{go}^2}{b} + T^2 t_{go} + \frac{2T^3 t_{go} + T^4}{c} - \frac{T^3}{2} + \frac{1}{a} \tag{19}$$

The feedback gains can now follow from the inversion of S and the use of Eq. (10). Because of the complexity of the results, this stage is better carried out with numerical values than with symbols.

For the interesting special case where $a \to \infty$, $b = c \to 0$ (i.e., game-theoretic perfect intercept without additional terminal constraints), we get

$$u = \frac{N'(x_1 + x_2 t_{go})}{t_{go}^2} - K_L x_3 \tag{20}$$

where

$$N' = \frac{6h^2(e^{-h} - 1 + h)}{2(1 - \gamma^{-2})h^3 + 3 + 6h - 6h^2 - 12he^{-h} - 3e^{-2h}} \tag{21}$$

$$K_L = \left(N'T^2 / t_{go}^2\right)(e^{-h} + h - 1) \tag{22}$$

and

$$h = t_{go}/T \tag{23}$$

which resembles Eqs. (3–6) for the case where $n_T = 0$ except for the additional term in the denominator, which converges to zero as $\gamma \to \infty$. Note also that for the case $T \to 0$ (i.e., $h \to \infty$), while $\gamma < \infty$, we obtain

$$u = \frac{3V_c\dot{\lambda}}{(1 - \gamma^{-2})} \tag{24}$$

where $\dot{\lambda}$ is given by Eq. (2). This is another well-known result.[8]

IV. Numerical Example

A. Interception Conflict Description, Guidance Laws, and Error Sources

In this section, we consider a numerical example that illustrates the merits of the reduced-sensitivity guidance. We analyze the effect on the miss distance of a 3-g constant acceleration target maneuver, a 3-g sinusoidal target maneuver of 2-rad/s frequency, and a 10-deg heading error. The conflict is assumed to take 3 s, and the time-constant T of Eq. (7) is taken to be 0.5 s. The conflict is assumed to be tail-chase or head-on with a pursuing velocity of 300 m/s. The effects of three guidance laws will be analyzed.

1) Minimum effort guidance, which includes a target acceleration feedback,

$$u_{\mathrm{MEL}}(t_{go}) = \frac{N'(x_1 + x_2 t_{go})}{t_{go}^2} - K_L x_3 + \frac{N'}{2}n_T \tag{25}$$

where N' and K_L are given by Eqs. (4–6).

2) Reduced sensitivity guidance law

$$u_{\mathrm{RSL}}(t_{go}) = \begin{bmatrix} 0 & 0 & -\frac{1}{T} \end{bmatrix} S^{-1}(t_{go}) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{26}$$

where S is given by Eqs. (14–19) with $a = 10{,}000$, $b = 1000$, $c = 1000$, and $\gamma = 12$. This value of γ is larger than the minimum γ, which still allows a bounded solution to Eq. (11) within $t \in [0, 3]$.

3) Proportional navigation

$$u_{\mathrm{PN}} = N'V_c\dot{\lambda} \tag{27}$$

with $N' = 3$. The line-of-sight rate $\dot{\lambda}$ of Eq. (2) is assumed to be exactly measured and is not affected by possible errors in the time-to-go estimation. Therefore, unlike the MEL and the RSL (reduced sensitivity guidance law), the performance of the PN (proportional navigation guidance law) will not depend on these errors.

Two possible types of errors can corrupt the time-to-go estimation. These are multiplicative (i.e., scale factor) and additive (i.e., bias or noise) errors. Namely,

$$t_{go}^{\text{measured}} = at_{go} + b + n \tag{28}$$

where a, b, and n denote the scale factor, bias error, and noise, respectively. The analysis of the effect of the noise error is out of the scope of the present paper. In the present paper, the results of a thorough analysis of the effect of bias errors b in the range of [0, 0.5] s will be described. Because of lack of space and to not burden the reader with many numerical results, the effects of scale-factor errors will be described to give an idea about the general trends. All of these guidance laws will be considered, namely, the MEL, RSL, and PN. Note that the MEL uses target acceleration measurements, whereas the RSL and PN do not. Therefore, to allow a meaningful comparison to the RSL, we also consider a variant of the MEL that does not use target acceleration measurements. Namely,

$$u_{\mathrm{MELN}}(t_{go}) = \frac{N'(x_1 + x_2 t_{go})}{t_{go}^2} - K_L x_3 \tag{29}$$

The effect on the interceptor's performance of a time-to-go estimation error, which varies between 0 and 0.5 s, is computed for the four guidance laws and all three error sources (target maneuvers and heading error). Thus, in Eqs. (25), (26), and (29) the measured time-to-go of Eq. (28) will be taken where a is 1 and n is zero. Comparison of the three guidance laws is made by looking at a few important variables: 1) miss distance and maximum commanded interceptor acceleration due to target maneuver and target acceleration; 2) miss distance and maximum commanded interceptor acceleration due to heading error.

In contrast to Ref. 4, we have assumed that the interceptor's time constant T is not subject to uncertainties. The situation we analyze may, therefore, correspond to a missile that applies a gain-scheduled autopilot allowing only insignificant variations of the closed-loop transfer function, which relates the acceleration command to the actual acceleration. We note that the PN guidance law was included in the comparison because it is well known and is widely applied in practice. However, the PN guidance law exhibits a divergence some time before the conflict end. Therefore, it may use very large accelerations that become unbounded at t_f, possibly resulting in a very small miss distance that may not be achieved in practice because of maneuver limits of the pursuer. To make the comparison to the other three guidance laws (namely, MEL, RSL, and MELN) more equitable, we chose to include in the simulations a hard limit of 15 g on the commanded target acceleration. This limit is reasonable, considering the 3-g maneuvers we take for the target (either of step or of sinusoidal type). Note, however, that the effect of this limit is not included in the results of the adjoint simulations because the adjoint analysis is based on a linear model of the closed loop. Adjoint simulations are frequently used in analyzing miss distances and are described in Refs. 2 and 9.

B. Method of Simulation and State-Feedback Gains Computation

To work out the example, we used a MATLAB™ program that includes both adjoint and forward simulations of the dynamics of Eq. (8). The adjoint simulation model was obtained by taking the state-space representation (see Ref. 9) of the adjoint of Eq. (8). The gains for the MEL, RSL, and MELN are computed by replacing the time-to-go in Eqs. (25), (26), and (29) by its measured value [Eq. (28)]: a is 1, n is zero, and b varies between 0 and 0.5. The PN gains are computed using Eqs. (27) and (2) where the exact value of the time-to-go is used. This situation represents the case where the PN law is mechanized by using a gimbaled seeker that directly measures the line-of-sight rate.

C. Simulation Results: Miss Distance and Maximum Accelerations

To allow for a clear comparison, we classify the simulation results according to their error sources (i.e., target acceleration step, sinusoidal target acceleration, and heading error). The most meaningful comparison is of the MELN, RSL, and PN guidance laws, which do not use target acceleration measurements. The MEL that uses target acceleration measurements is also included in the comparison just for reference and to see how acceleration measurements can improve performance when the target maneuvers. The results are illustrated by a comparison of the graphs for the four guidance laws.

1. *Step of 3 g in Target Acceleration*

The RSL law is shown in Fig. 1 to have an obvious lower sensitivity to the time-to-go estimation error than the MELN with considerably lower miss-distance values. The PN is completely insensitive to the time-to-go estimation error and exhibits miss-distance values that are even smaller than those of the RSL. Thus, it may seem at first sight that if acceleration measurements are not available, the best choice is to use the PN guidance law. This impression appears to be wrong if the maximum accelerations used throughout the interception conflict are compared (Fig. 2). This comparison shows that the PN uses more than twice as much acceleration as the RSL. The MELN uses accelerations almost as large as the PN but with a rather poor miss distance. Thus, the RSL seems to be the best choice if both miss-distance and interceptor maneuvers are taken into account, and target acceleration measurements are not available. If, however, target-acceleration measurements are available, the MEL has an advantage over RSL in terms of both miss distance and the required maneuvers. The nominal performance (i.e., with zero error on the time-to-go) could be expected, because the MEL is especially tailored to constant target accelerations. What is surprising is its relative insensitivity to time-to-estimation errors. The best choice when target acceleration measurements are available, therefore, is the MEL. However, if these measurements are not perfect (and they never are), the sensitivity to the measurement errors should be kept in mind and computed before finally deciding to use MEL. The above discussion does not change much if the adjoint simulation results of Fig. 3 are considered. These results were obtained for a fixed time-to-go error of 0.2 s, where the effect of acceleration limit was ignored.

2. *A 3-g, 2-rad/s Sinusoidal Acceleration*

With this type of target maneuver, the PN is no longer an adequate choice. It has poor miss-distance performance and requires more interceptor maneuvers than the RSL and MELN. Comparing the RSL and MELN (Fig. 4) shows in time-to-go errors, which are less

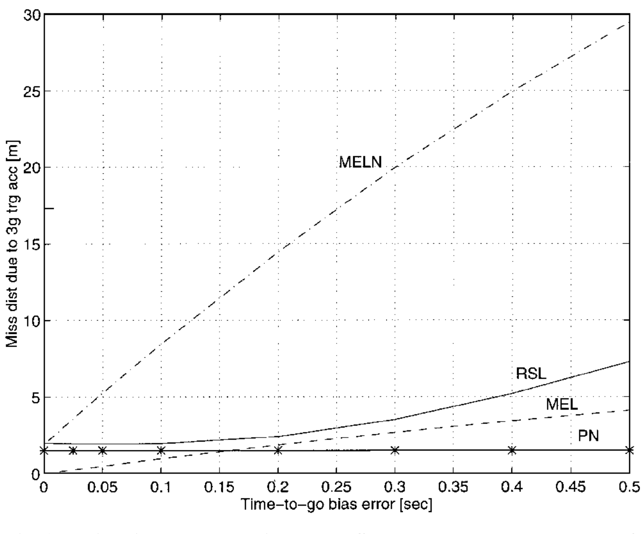

Fig. 1 Miss distance—case A: ——, RSL; – – –, MEL; – · –, MEL–NO Trg. acc. feedback; and ∗ ∗ ∗, PN.

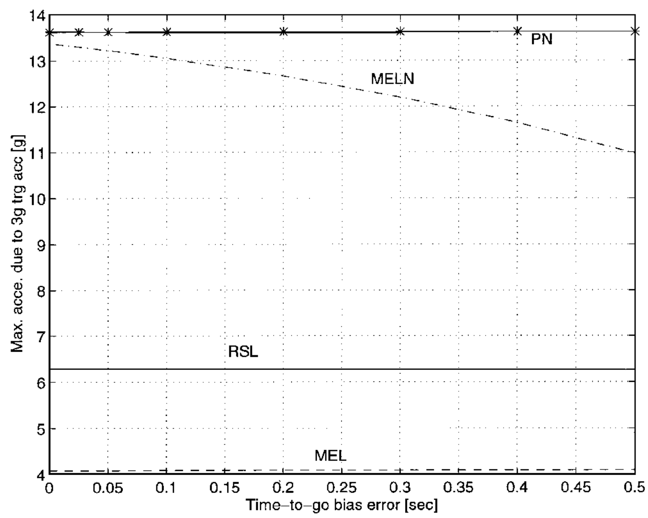

Fig. 2 Maximum commanded acceleration—case A: ——, RSL; – – –, MEL; – · –, MEL–NO Trg. acc. feedback; and ∗ ∗ ∗, PN.

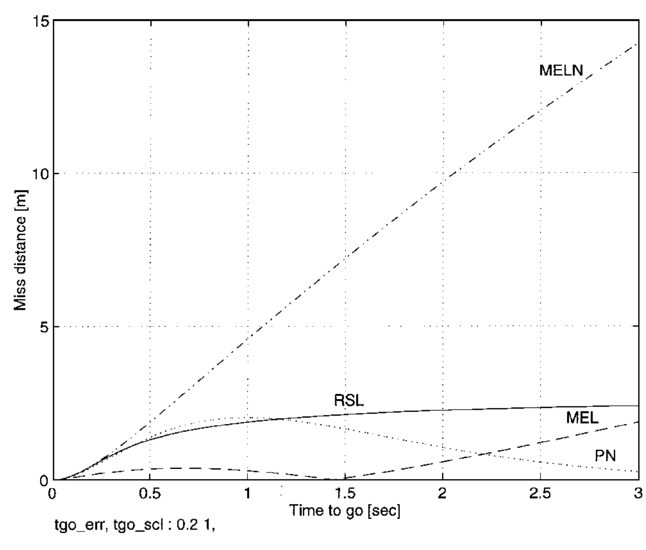

Fig. 3 Miss distance—case A (adjoint simulation): 3 g target acceleration.

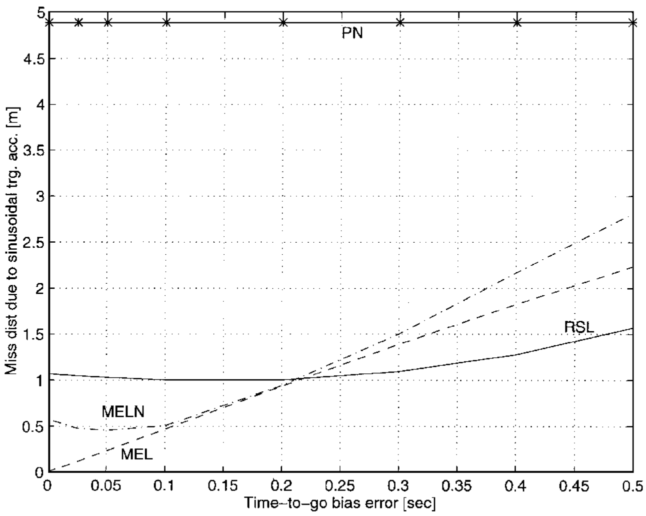

Fig. 4 Miss distance—case B: ——, RSL; - - -, MEL; - · -, MEL–NO Trg. acc. feedback; and * * *, PN.

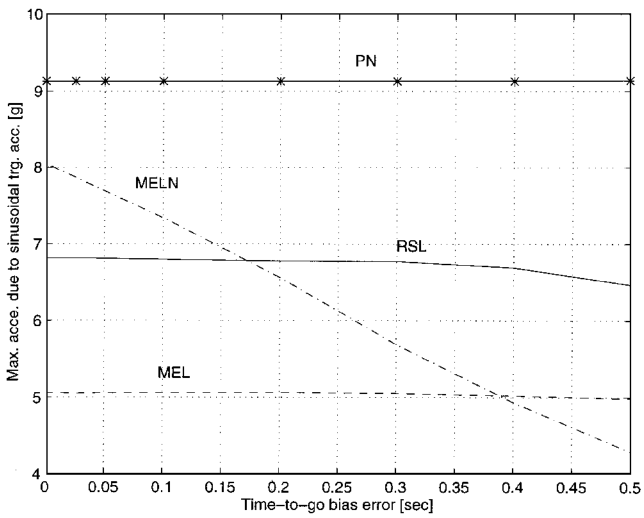

Fig. 5 Maximum commanded acceleration—case B: ——, RSL; - - -, MEL; - · -, MEL–NO Trg. acc. feedback; and * * *, PN.

than about 0.1 s, that the MELN has a somewhat lower miss distance than the RSL (about 0.5 and 1 m, respectively), with the MELN requiring more maneuvers (8 and 7 g, respectively; Fig. 5). However, in time-to-go errors that are larger than about 0.2 s, the RSL provides a lower miss distance at the cost of larger required maneuvers. This superiority is maintained also for larger values of the error. Clearly, RSL is significantly less sensitive to time-to-go errors. Note that MEL is also less sensitive than MELN to time-to-go estimation errors and is comparable to RSL (40% less acceleration required with somewhat inferior miss distances).

Remark. With sinusoidal maneuvers, the frequency and the initial phase of the maneuver function affect the results. We have chosen to deal with a single frequency, and, therefore, our conclusions about sinusoidal maneuvers should be used carefully. We chose to overcome the phase sensitivity of the results by taking phases within [−180, 180] deg with 45-deg steps, and the results shown are the average values.

3. *Heading Error of 10 deg*

This is where the RSL performance is the most impressive. The MELN, which coincides in this case with the MEL, has a miss distance that is almost linear with the time-to-go estimation error, with about an 8-m miss distance for an estimation error of 0.2 s, and is in fact worse than PN for estimation errors higher than 0.03 s. The RSL, on the other hand, shows a very low sensitivity that leads to about a 0.3-m miss distance for a time-to-go estimation error of 0.2 s (Fig. 6) and less than 2 m with errors as large as 0.5 s. The

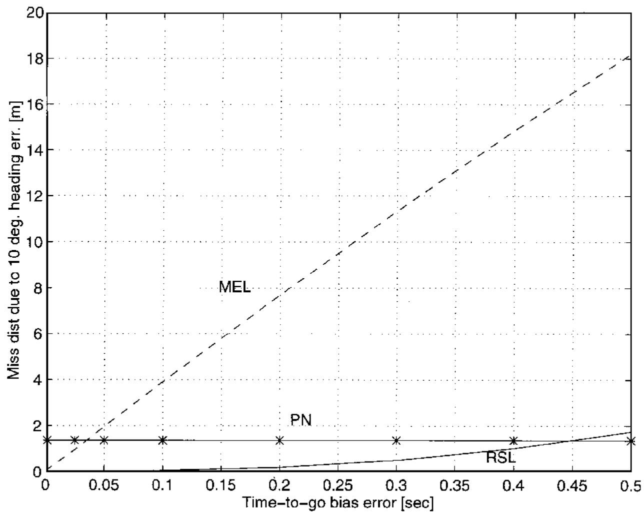

Fig. 6 Miss distance—case C: ——, RSL; - - -, MEL; and * * *, PN.

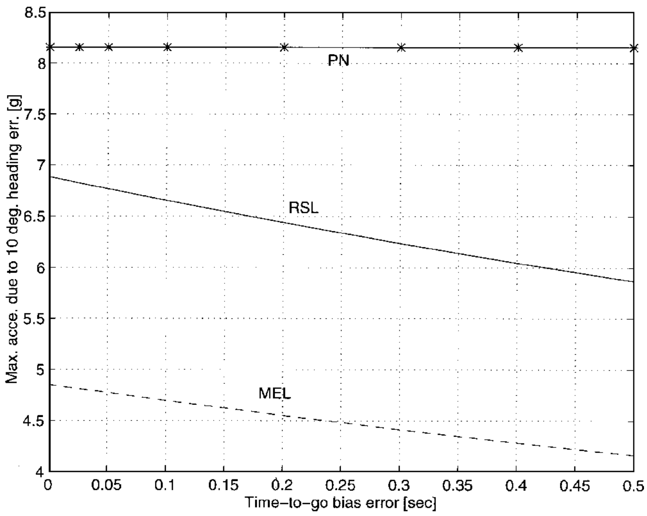

Fig. 7 Maximum commanded acceleration—case C: ——, RSL; - - -, MEL; and * * *, PN.

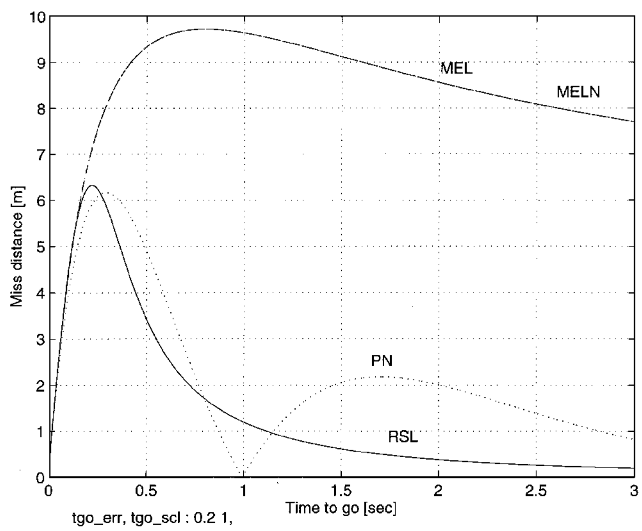

Fig. 8 Miss distance—case C (adjoint simulation): heading error of 10 deg.

required accelerations are still larger with the RSL (Fig. 7). Also for short-duration interceptions (as short as about 1 s), the superiority of the RSL (in terms of miss distances) is maintained. This is shown by the adjoint simulations results for this case, which are depicted in Fig. 8 (for a time-to-go error of 0.2 s).

Remark. Note that we considered only positive bias errors on the time-to-go. Because at zero time-to-go, the MEL and MELN gains become unbounded, negative bias errors bring this singular point into the interception conflict time interval [see Eq. (28) and consider negative b]. Thus, the commanded accelerations go unbounded. To overcome this difficulty, a number of policies can be adopted:

1) A fixed positive bias, which is larger in its magnitude than the largest possible bias on Eq. (28), can be added. In such a case, the nominal performance will be degraded, but divergence will be prevented.

2) The guidance gains can be limited, i.e., above certain values they will be frozen.

3) Negative values of the estimated time-to-go-error can be avoided (i.e., set to zero if negative estimation is obtained).

4) Acceleration command can be frozen above a certain limit.

The performance of all guidance laws (except for PN, which is insensitive), is obviously policy-dependent for negative errors and will not be analyzed in the present paper.

Remark. As we have already mentioned, another relevant type of error in the time-to-go estimation is the one of scale factor. We have also conducted simulations for this type of error. For heading errors, the general trends are very much like those with bias-type errors. Namely, the RSL is clearly superior over MEL/MELN. It is also better than the PN (unlike the case of bias-type time-to-go errors). For constant acceleration maneuvering targets, the RSL still performs better than the MELN and worse than the MEL. For sinusoidal maneuvering targets, the results are equivocal. To obtain a complete comparison, different frequencies should be checked. The numerical results for the effect of scale factors on the performance are not included.

V. Conclusions

An optimal guidance law that reduces the effect of the estimation error of the time-to-go was developed. This guidance scheme is recommended for cases in which target maneuvers are insignificant (e.g., stationary targets) or when the measurement of the target acceleration is not available. In such cases, the reduced-sensitivity guidance law of the present paper outperforms the minimum-effort guidance law and, for reasonable time-to-go errors, is better than the PN guidance law when mechanized by line-of-sight rate measurement (i.e., inherently insensitive to time-to-go estimation errors).

The derivation applies the solution of a standard linear quadratic differential game and leads to closed-form formulas for the inverse of the corresponding Riccati equation. The solution does not use an estimation of the target's acceleration. However, the idea of the present paper may be generalized to guidance systems that contain this important measurement whenever it is available. One way to achieve this is to consider a full-information pattern for the differential game rather than just a full-state information pattern. Another possible way is to augment the state-space model of the pursuit–evasion game to include an additional state that corresponds to the target's acceleration. Then the problem will still be within the framework of a state-feedback (rather than full-information) pattern, but the increased system order may pose difficulties in obtaining closed-form formulae. This issue is the subject of future research.

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